# Solutions Week 6.

1) Qubit measurement

$$|\psi\rangle = \cos 0.35\, |0\rangle + e^{2\pi i\, 0.75} \sin 0.35\, |1\rangle$$

a) $\langle Z \rangle$ observable

two ways how to tell what we measure

I) $P_0 = |\cos 0.35|^2 = \cos^2 0.35$

$P_1 = \cancel{|\sin 0.35|^2} = |e^{2\pi i\, 0.75} \sin 0.35|^2 = \sin^2 0.35$

II) expected average value

$E(Z) = \langle Z \rangle = \langle \psi | Z | \psi \rangle =$

$$\begin{pmatrix} \cos 0.35 \\ e^{2\pi i} \sin 0.35 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \boxed{\begin{matrix} \cos 0.35 \\ -e^{2\pi i\, 0.75} \cdot \sin 0.35 \end{matrix}}$$

$$\left(\cos 0.35,\ e^{-2\pi i} \sin 0.35\right) \quad \cos^2 0.35 - \sin^2 0.35$$

$$\Rightarrow \boxed{E(Z) = \cos^2 0.35 - \sin^2 0.35 = P_0 - P_1}$$

which also agrees with the definition of average value
$\Downarrow$
sum of (results times their respective probabilities)

$$E(M) = \sum \lambda_m \, P(\lambda_m)$$

some general observable — eigvals of $M$ — probability to measure a particular eigval of $M$

$$\Rightarrow E(Z) = (1) \cdot \underbrace{P(1)}_{P_0} + (-1) \underbrace{P(-1)}_{P_1}$$

$$= P_0 - P_1$$

If physicists measures qubits, ... $E(M)$ is what he will give you.

We did not get any information about the phase!

From $E(Z)$ we can easily compute

e.g. $P_0$, since $P_0 + P_1 = 1$

$$\Rightarrow M(Z) = P_0 - P_1 = P_0 - (1 - P_0) = 2P_0 - 1$$

$$P_0 = \frac{M(Z) + 1}{2}$$