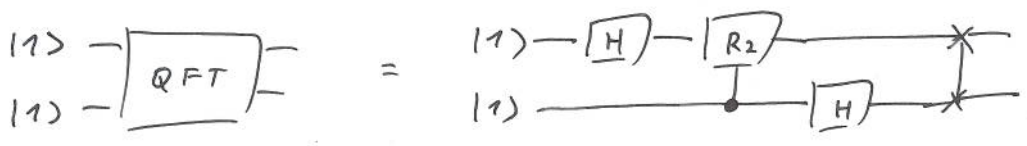


$$c) \text{QFT}^{(2)} |3\rangle = \frac{1}{\sqrt{2^2}} \left(e^{2\pi i 3 \cdot 0/4} |0\rangle + e^{2\pi i 3 \cdot 1/4} |1\rangle + e^{2\pi i 3 \cdot 2/4} |2\rangle + e^{2\pi i 3 \cdot 3/4} |3\rangle \right)$$

$$= \frac{1}{2} \left(|0\rangle - i |1\rangle - |2\rangle + i |3\rangle \right)$$



$$|1\rangle |1\rangle \xrightarrow{H \otimes I} \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) |1\rangle \xrightarrow{\text{CNOT-R}_2} \frac{1}{\sqrt{2}} (|0\rangle - e^{2\pi i/4} |1\rangle) |1\rangle$$

$$\xrightarrow{I \otimes H} \frac{1}{2} (|0\rangle - e^{2\pi i/4} |1\rangle) (|0\rangle - |1\rangle)$$

$$= \frac{1}{2} (|00\rangle - e^{2\pi i/4} |10\rangle - |01\rangle + e^{2\pi i/4} |11\rangle)$$

$$\xrightarrow{\text{swap}} \frac{1}{2} (|00\rangle - e^{2\pi i/4} |01\rangle - |10\rangle + e^{2\pi i/4} |11\rangle)$$

$$= \frac{1}{2} (|0\rangle - e^{2\pi i/4} |1\rangle - |2\rangle + e^{2\pi i/4} |3\rangle)$$

$$= \frac{1}{2} (|0\rangle - i |1\rangle - |2\rangle + i |3\rangle)$$

2) Grover

$$a) H (2|0\rangle\langle 0| - I) H = H \left(\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) H$$

$$= H \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} H = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = X \quad (\text{or NOT})$$

thus, in the single qubit domain, inversion about average = X

$$b) \text{INV}^{(1)} (d_0 |0\rangle + d_1 |1\rangle) = (2\langle d \rangle - d_0) |0\rangle + (2\langle d \rangle - d_1) |1\rangle$$

$$= \left(2 \cdot \frac{d_0 + d_1}{2} - d_0 \right) |0\rangle + \left(2 \cdot \frac{d_0 + d_1}{2} - d_1 \right) |1\rangle$$

$$= d_1 |0\rangle + d_0 |1\rangle = d_1 X |1\rangle + d_0 X |0\rangle = X (d_0 |0\rangle + d_1 |1\rangle)$$

$$c) \frac{1}{2} \xrightarrow{\text{V}_f} \frac{1}{2} \xrightarrow{\text{INV}} 1 \rightarrow$$

$2\langle d \rangle = 2 \cdot \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2} - \frac{1}{2} \right) / 2 = \frac{1}{2}$
 $2\langle d \rangle - \frac{1}{2} = 0$ I can search 1-out-of-4
 $2\langle d \rangle + \frac{1}{2} = 1$ items exactly in 1 step