Solutions. Week 5.

1) Quantum Fourier transform

a)

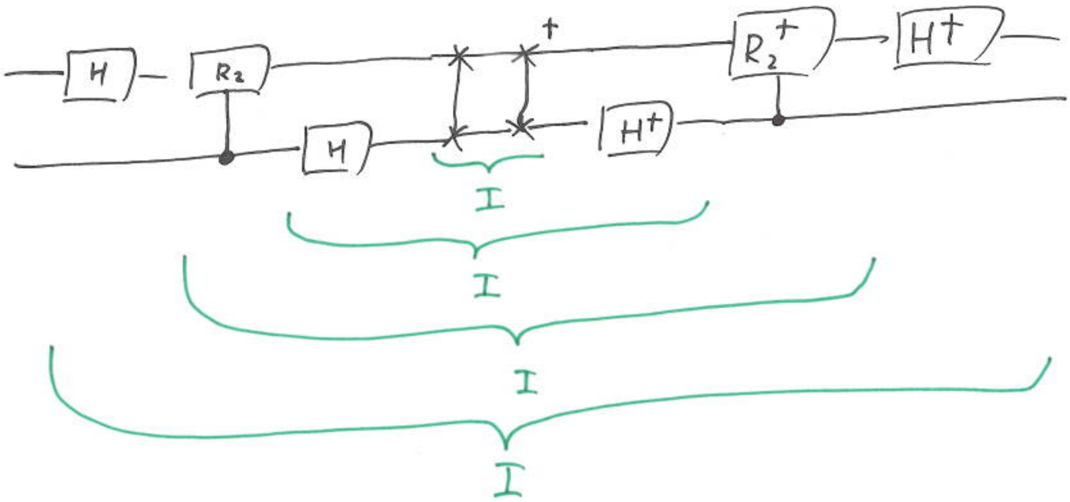
$$QFT^{(1)}|0\rangle = \frac{1}{\sqrt{2}}\left(\underbrace{e^{2\pi i\, 0.0/2}}_{1}|0\rangle + \underbrace{e^{2\pi i\, 0.1/2}}_{1}|1\rangle\right)$$

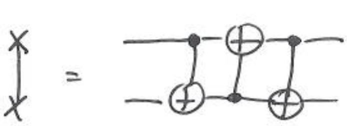
$$= \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = H|0\rangle$$

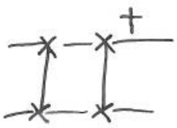
$$QFT^{(1)}|1\rangle = \frac{1}{\sqrt{2}}\left(\underbrace{e^{2\pi i\, 1.0/2}}_{1}|0\rangle + \underbrace{e^{2\pi i\, 1.1/2}}_{-1}|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = H|1\rangle$$

b)



1) 



reduces to showing

$$CNOT \cdot CNOT^{\dagger} = I$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix} \cdot \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

2) $H = \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$, $H^{\dagger} = \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$

$$H \cdot H^{\dagger} = I$$

3) $R_2 = \begin{pmatrix}1&0\\0&e^{2\pi i/4}\end{pmatrix}$, $R_2^{\dagger} = \begin{pmatrix}1&0\\0&e^{-2\pi i/4}\end{pmatrix}$ $\qquad R_2 \cdot R_2^{\dagger} = I$

4) $H \cdot H^{\dagger} = I$