2) Partial trace

a) $$S_A = \begin{pmatrix} x_{0,0} + x_{1,1} & x_{0,2} + x_{1,3} \\ x_{2,0} + x_{3,1} & x_{2,2} + x_{3,3} \end{pmatrix}$$

b) $$|\psi\rangle\langle\psi| = \frac{1}{2}\left(|00\rangle + |11\rangle\right)\left(\langle 00| + \langle 11|\right)$$

$$= \frac{1}{2}\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right)$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \quad S_A = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

maximally mixed state

c) ~~[illegible]~~

$$|\psi\rangle' = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)|0\rangle + \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)|1\rangle\right)$$

$$= \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle - |11\rangle\right)$$

$$|\psi'\rangle\langle\psi|' = \frac{1}{4}\left(|00\rangle + |01\rangle + |10\rangle - |11\rangle\right)\left(\langle 00| + \langle 01| + \langle 10| - \langle 11|\right)$$

$$= \frac{1}{4}\Big(|00\rangle\langle 00| + |00\rangle\langle 01| + |00\rangle\langle 10| - |00\rangle\langle 11|$$
$$+ |01\rangle\langle 00| + |01\rangle\langle 01| + |01\rangle\langle 10| - |01\rangle\langle 11|$$
$$+ |10\rangle\langle 00| + |10\rangle\langle 01| + |10\rangle\langle 10| - |10\rangle\langle 11|$$
$$- |11\rangle\langle 00| - |11\rangle\langle 01| - |11\rangle\langle 10| + |11\rangle\langle 11|\Big)$$

$$= \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{pmatrix}$$

$$S_A = \frac{1}{4}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

max mix again