Solutions Week 4.

1) Density matrices

a) $|0\rangle\langle 0|$ $\quad S = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $\quad P_0 = 1 \quad P_1 = 0$

b) $|1\rangle\langle 1|$ $\quad S = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ $\quad P_0 = 0 \quad P_1 = 1$

c) $\frac{1}{2}(|0\rangle + |1\rangle)(\langle 0| + \langle 1|) = \frac{1}{2}(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|)$

$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $\quad P_0 = \frac{1}{2}$, $P_1 = \frac{1}{2}$

d) $\frac{1}{2}(|0\rangle - |1\rangle)(\langle 0| - \langle 1|) = \frac{1}{2}(|0\rangle\langle 0| - |0\rangle\langle 1| - |1\rangle\langle 0| + |1\rangle\langle 1|)$

$= \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ $\quad P_0 = \frac{1}{2}$, $P_1 = \frac{1}{2}$

e) $0.4\,|0\rangle\langle 0| + 0.6\,|1\rangle\langle 1| = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.6 \end{pmatrix}$ $\quad P_0 = 0.4$, $P_1 = 0.6$

f) $0.3\,|0\rangle\langle 0| + 0.7 \cdot \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0.65 & 0.35 \\ 0.35 & 0.35 \end{pmatrix}$ $\quad P_0 = 0.65$, $P_1 = 0.35$

g) $0.3\,|0\rangle\langle 0| + 0.7\,(\sqrt{0.2}\,|0\rangle + \sqrt{0.8}\,|1\rangle)(\sqrt{0.2}\,\langle 0| + \sqrt{0.8}\,\langle 1|)$

$= \begin{pmatrix} 0.3 & 0 \\ 0 & 0 \end{pmatrix} + 0.7\,(0.2\,|0\rangle\langle 0| + \sqrt{0.2 \cdot 0.8}\,|0\rangle\langle 1| + \sqrt{0.2 \cdot 0.8}\,|1\rangle\langle 0| + 0.8\,|1\rangle\langle 1|)$

$= \begin{pmatrix} 0.44 & 0.28 \\ 0.28 & 0.56 \end{pmatrix}$ $\quad P_0 = 0.44$, $P_1 = 0.56$

h) $0.2\,|0\rangle\langle 0| + 0.1\,|1\rangle\langle 1| + \frac{0.3}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \frac{0.4}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 0.2 + 0.15 + 0.2 & 0.15 - 0.2 \\ 0.15 - 0.2 & 0.1 + 0.15 + 0.2 \end{pmatrix} = \begin{pmatrix} 0.55 & -0.05 \\ -0.05 & 0.45 \end{pmatrix}$ $\quad P_0 = 0.55$, $P_1 = 0.45$