

3) Bell inequality

Week 3.

(2)

a) Eq. 2.225

We need two properties of the mean value

$$E(\vec{X}) = \sum_i E(X_i)$$

 \vec{X} a vector of results X_i

$$E(\vec{X}) = \sum_i P_i X_i$$

 P_i a probability of result X_i

and we apply them to the quantity $(QS + RS + RT - QT)$.
Of course, the main difficulty is to design this quantity....

b) Eq. 2.230

for example, $\langle QS \rangle$

$$QS = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$\text{for } |\psi\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

we get

$$\langle QS \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} (0 \ 1 \ -1 \ 1) \boxed{\frac{1}{\sqrt{2}}}$$