

1) Projective measurements

The standard observable $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Average values? (Denoted $E(Z)$ or $\langle Z \rangle$)

a) $|0\rangle$ $\langle Z \rangle = \langle 0|Z|0\rangle = \langle 0|0\rangle = 1$

b) $|1\rangle$ $\langle Z \rangle = \langle 1|Z|1\rangle = -\langle 1|1\rangle = -1$

c) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\langle Z \rangle = \frac{1}{\sqrt{2}}(\langle 0| + \langle 1|)Z \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
 $= \frac{1}{2}(\langle 0| + \langle 1|)(|0\rangle - |1\rangle)$
 $= \frac{1}{2} \begin{pmatrix} 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0$

2) Observables $\vec{v} \cdot \vec{\sigma}$

I) Observable $M = X$, $E(M) = ?$

a) $|0\rangle$ $\langle X \rangle = \langle 0|X|0\rangle = \langle 0|1\rangle = 0$

b) $|1\rangle$ $\langle X \rangle = \langle 1|X|1\rangle = \langle 1|0\rangle = 0$

c) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\langle X \rangle = \frac{1}{\sqrt{2}}(\langle 0| + \langle 1|)X \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
 $= \frac{1}{2}(\langle 0| + \langle 1|)(|0\rangle + |1\rangle) = \frac{1}{2} \begin{pmatrix} 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1$

II) Observable $M = \frac{-Z - X}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$

a) $|0\rangle$ $\langle 0|M|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ -1 \end{pmatrix}$
 $(1 \ 0) \cdot \begin{pmatrix} -1 \\ -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}$

b) $|1\rangle$ $\langle 1|M|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ +1 \end{pmatrix}$
 $(0 \ 1) \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}$

c) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
 $\langle M \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}$
 $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -1 \end{pmatrix}$