

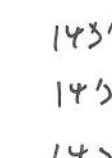


1) Quantum measurement

$$|4\rangle = \frac{1}{\sqrt{3}} (|1000\rangle + |1010\rangle + |1011\rangle)$$

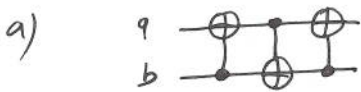
a)  $|4\rangle$ is not modified

b)  $\begin{cases} |4\rangle' = |1000\rangle & \text{with } p = \frac{1}{3} \\ |4\rangle' = \frac{1}{\sqrt{2}} (|1010\rangle + |1011\rangle) & \text{with } p = \frac{2}{3} \end{cases}$

c)  $\begin{aligned} |4\rangle' &= |1000\rangle & \text{with } p &= \frac{1}{3} \\ |4\rangle' &= |1010\rangle & \text{with } p &= \frac{1}{3} \\ |4\rangle' &= |1011\rangle & \text{with } p &= \frac{1}{3} \end{aligned}$

2) The Swap operation

$a, b \in \{0, 1\}$



$$|a, b\rangle \xrightarrow{1)} |a \oplus b, b\rangle \xrightarrow{2)} |a \oplus b, b \oplus (a \oplus b)\rangle = |a \oplus b, a\rangle \xrightarrow{3)} |(a \oplus b) \oplus a, a\rangle = |b, a\rangle$$

b) $\frac{1}{\sqrt{2}} (|100\rangle + |101\rangle) \xrightarrow{\text{swap}} \frac{1}{\sqrt{2}} (|100\rangle + |110\rangle)$

3) The Toffoli gate



$a, b, c \in \{0, 1\}$

a) $|1000\rangle \rightarrow |1000\rangle$

b) $|1110\rangle \rightarrow |1111\rangle$

c) $\frac{1}{\sqrt{2}} (|100\rangle + |101\rangle) \otimes |10\rangle \rightarrow \frac{1}{\sqrt{2}} (|1000\rangle + |1010\rangle) = \frac{1}{\sqrt{2}} (|100\rangle + |101\rangle) \otimes |10\rangle$