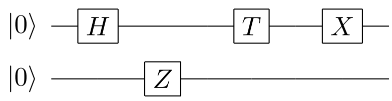
1 Normalization, relative phases, outcome probabilities

1. Normalize these vectors
   - $(4+3i)\left|0\right\rangle + \frac{e^{-i\pi/8}}{\sqrt{2}}\left|1\right\rangle \Rightarrow \frac{1}{\sqrt{25.5}}\left((4+3i)\left|0\right\rangle + \frac{e^{-i\pi/8}}{\sqrt{2}}\left|1\right\rangle\right)$
   - $2^3\left|0\right\rangle - i\left|1\right\rangle \Rightarrow \frac{1}{\sqrt{65}}\left(2^3\left|0\right\rangle - i\left|1\right\rangle\right)$
2. Calculate relative phases
   - $\frac{1}{\sqrt{25.5}}(4+3i)\left|0\right\rangle + \frac{e^{-i\pi/8}}{\sqrt{2}}\left|1\right\rangle = e^{i\arctan(3/4)}\left(\sqrt{\frac{4^2+3^2}{25.5}}\left|0\right\rangle + \frac{e^{-i(\pi/8+\arctan(3/4))}}{\sqrt{51}}\left|1\right\rangle\right)$, thus the relative phase is $e^{-i(\pi/8+\arctan(3/4))}$.
   - The relative phase is $-i$.
3. Calculate outcome probabilities
   - $P_{\left|0\right\rangle} = \frac{25}{25.5}$, $P_{\left|1\right\rangle} = \frac{1}{51}$. It holds that $P_{\left|0\right\rangle} + P_{\left|1\right\rangle} = 1$.
   - $P_{\left|0\right\rangle} = \frac{(2^3)^2}{65}$, $P_{\left|1\right\rangle} = \frac{1}{65}$. It holds that $P_{\left|0\right\rangle} + P_{\left|1\right\rangle} = 1$.

2 Tensor products

- $\left|0\right\rangle \otimes \frac{1}{\sqrt{2}}(\left|0\right\rangle + \left|1\right\rangle) = \frac{1}{\sqrt{2}}(\left|00\right\rangle + \left|01\right\rangle)$
- $\left(\frac{1}{\sqrt{3}}\left|0\right\rangle + e^{i\pi/4}\sqrt{\frac{2}{3}}\left|1\right\rangle\right) \otimes \frac{1}{\sqrt{2}}(\left|0\right\rangle + \left|1\right\rangle)$
  $= \frac{1}{\sqrt{6}}\left|00\right\rangle + \frac{1}{\sqrt{6}}\left|01\right\rangle + \frac{e^{i\pi/4}}{\sqrt{3}}\left|10\right\rangle + \frac{e^{i\pi/4}}{\sqrt{3}}\left|11\right\rangle$
- $H \otimes I = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$
- $Z \otimes X = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$
- $X \otimes H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$

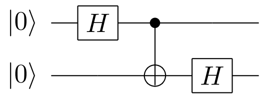
3 Simple circuit



1. What is the overall state of both qubits?
   $\frac{1}{\sqrt{2}}\left(e^{i\pi/4}|00\rangle + |10\rangle\right)$

2. Is it a product or entangled state?
   It is a product state equal to
   $\frac{1}{\sqrt{2}}\left(e^{i\pi/4}|0\rangle + |1\rangle\right) \otimes |0\rangle$

4 Circuit with CNOT



- What is the overall state?
  $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$

- How does it differ when the CNOT gate is omitted?
  We get $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$,
  which is a product state equal to
  $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.

5 Gate definitions

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$