Problems Week 6.

1) Qubit measurement

There is a source of qubits producing

$$|\psi\rangle = \cos(0.35)|0\rangle + e^{2\pi i \, 0.75} \sin(0.35)|1\rangle$$

a) What kind of information about $|\psi\rangle$ do you acquire after $N = 100$ measurements of the std. observable $\langle Z \rangle$ ($\Rightarrow$ projections on the states $|0\rangle$ and $|1\rangle$, the eigenvectors of Z)

Hint: the number N here gives you the accuracy
$\Rightarrow$ from the central limit theorem
from N measurements you get accuracy of order $\frac{1}{\sqrt{N}}$ after N measurements

b) What kind of information about $|\psi\rangle$ do you acquire by measuring the observable $\langle X \rangle$?

Hint: observable $\langle X \rangle$: projection onto the states

$$\left.\begin{aligned} |+\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\ |-\rangle &= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \end{aligned}\right\} \text{eigenvectors of } X$$

since:

$$|0\rangle \overset{H}{\longleftrightarrow} |+\rangle$$
$$|1\rangle \overset{H}{\longleftrightarrow} |-\rangle$$

We can easily implement measurement of $\langle X \rangle$ by performing the H gate and then project onto $|0\rangle$ and $|1\rangle$ in a std. way.

c) Combine results from a) and b) to describe your knowledge of $|\psi\rangle$