c) Verify that the circuit for QFT over 2 qubits can be desribed by

$$QFT\,|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} e^{2\pi i x y / 2^n} |y\rangle .$$

Verify this for $x = 3$ ($\Rightarrow |11\rangle$ in binary), $n=2$

2) Grover

The operation "Inversion about average" can be desribed in a matrix form as (over n qubits)

$$H^{(n)} \left(2|0\rangle\langle 0|^{(n)} - I^{(n)} \right) H^{(n)} .$$

a) Find a circuit for $n=1$, make as simple as possible.

b) make sure that it takes $\sum \alpha_x |x\rangle$ into $\sum (2\langle\alpha\rangle - \alpha_x)|x\rangle$

i.e. ~~that~~ in the single qubit domain:

$$\alpha_0|0\rangle + \alpha_1|1\rangle \rightarrow (2\langle\alpha\rangle - \alpha_0)|0\rangle + (2\langle\alpha\rangle - \alpha_1)|1\rangle$$

$$\langle\alpha\rangle = \frac{\alpha_0 + \alpha_1}{2}$$

c) Consider $x = \{0, 1, 2, 3\}$ and exactly one of those x's satisfies $f(x) = 1$ for some function f.

Run Grover search. How many times do we have to invoke V_f and inversion about the average? How many times <u>exactly</u>?