Problems Week 5.

1) Quantum Fourier transform

$$QFT\,|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} e^{2\pi i\, x y / 2^n} \,|y\rangle$$

a) Verify that the QFT over 1 qubit is implemented by a single Hadamard gate

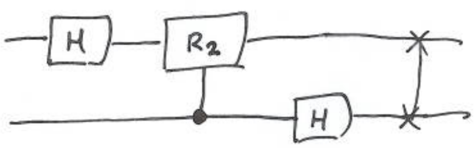
$$H\,|0\rangle = \frac{1}{\sqrt{2^1}} \sum_{y=0}^{2^1-1} e^{2\pi i \ldots} \quad \ldots$$

$$H\,|1\rangle = \ldots$$

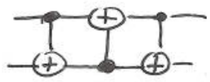
b) Verify that the following circuit for QFT over 2 qubits is a unitary operation, i.e. $QFT \cdot QFT^\dagger = I$

$\dagger$ = transpose and complex conjugate

$QFT^{(2)}$:



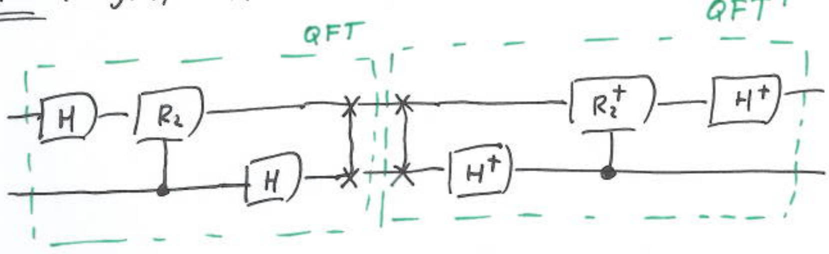
is a swap gate



$$R_k = \begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^k} \end{pmatrix}$$

$$R_2 = \ldots$$

hint: graphical intuition



now, two swaps should obviously give identity

$H^\dagger = ?$, $H H^\dagger = ?$, $R_2^\dagger = ?$, ...