1) Density matrix

def: $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

A density matrix contains all information we can potentially obtain from the system.
In particular, probabilities of observing the state $|0\rangle$ resp $|1\rangle$ upon measurement lay on the diagonal.

Write down corresponding density matrices and probabilities for $|0\rangle$ resp $|1\rangle$ as results of the measurement.

a) $|0\rangle$
b) $|1\rangle$
c) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
d) $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
e) $0.4\,|0\rangle + 0.6\,|1\rangle$
f) $0.3\,|0\rangle + 0.7\,\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
g) $0.3\,|0\rangle + 0.7\,(\sqrt{0.2}\,|0\rangle + \sqrt{0.8}\,|1\rangle)$
h) $0.2\,|0\rangle + 0.1\,|1\rangle + 0.3\,\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + 0.4\,\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

2) Partial trace

def: $tr_B(|a_1, b_1\rangle\langle a_2, b_2|) = |a_1\rangle\langle a_2| \cdot \underbrace{\langle b_2|b_1\rangle}_{\text{a number}}$

... we traced out system B

a trace out of a system A would be

$$tr_A(|a_1, b_1\rangle\langle a_2, b_2|) = \langle a_2|a_1\rangle \cdot |b_1\rangle\langle b_2|$$

Consider a density matrix of a two qubit system

$$\rho_{AB} = \begin{pmatrix} X_{0,0} & X_{0,1} & X_{0,2} & X_{0,3} \\ X_{1,0} & X_{1,1} & X_{1,2} & X_{1,3} \\ X_{2,0} & X_{2,1} & X_{2,2} & X_{2,3} \\ X_{3,0} & X_{3,1} & X_{3,2} & X_{3,3} \end{pmatrix} = X_{0,0}|0\rangle\langle 0| + X_{0,1}|0\rangle\langle 1| + \dots + X_{1,0}|1\rangle\langle 0| + \dots + X_{3,3}|3\rangle\langle 3|$$

labelled in decimal system