1) Projective measurements

We have already described projective measurements in terms of an observable, which is a Hermitian operator.

Observable has a spectral decomposition:

$$M = \sum_m \lambda_m |m\rangle\langle m|$$

where λ_m are eigenvalues and $P_m = |m\rangle\langle m|$ are projectors.

Upon measuring the state $|\psi\rangle$, the probability to observe the "result" λ_m is given by

$$p(\lambda_m) = \langle\psi| P_m |\psi\rangle$$

The average value of the measurement is then

$$\begin{aligned} E(M) &= \sum \lambda_m p(\lambda_m) \\ &= \sum \lambda_m \langle\psi| P_m |\psi\rangle \\ &= \langle\psi| \sum \lambda_m P_m |\psi\rangle \\ &= \langle\psi| M |\psi\rangle . \end{aligned}$$

1) Suppose the standard observable $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ with eigenvalues ± 1. What is the average value of the measurement for the states

a) $|0\rangle$

b) $|1\rangle$

c) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. Should be zero, right?